

Exercise 63

Draw a diagram to show that there are two tangent lines to the parabola $y = x^2$ that pass through the point $(0, -4)$. Find the coordinates of the points where these tangent lines intersect the parabola.

Solution

The equation of a line is

$$y = mx + b.$$

Use the fact that the line passes through the point $(0, -4)$ to find b .

$$-4 = m(0) + b \quad \rightarrow \quad b = -4$$

The line that passes through $(0, -4)$ is then

$$y = mx - 4.$$

For this line to be tangent to the parabola, it has to intersect the parabola at exactly one point. Set the formulas equal to each other and then solve for x .

$$\begin{aligned} mx - 4 &= x^2 \\ x^2 - mx + 4 &= 0 \\ x &= \frac{m \pm \sqrt{m^2 - 16}}{2} \end{aligned}$$

For there to be only one intersection point, it must be that

$$m^2 - 16 = 0,$$

which means $m = \pm 4$. The two tangent lines to the parabola $y = x^2$ that pass through $(0, -4)$ are therefore

$$y + 4 = -4(x - 0) \quad \text{and} \quad y + 4 = 4(x - 0).$$

